Coffee Berry Borer (*Hypothenemus hampei*) and its role in the evolutionary diversification of the coffee market

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1. Introduction

On the one hand, coffee production is drastically affected by the CBB, a beetle that starts the destruction when it drills a coffee bean through the navel to the endosperm. It makes galleries to deposit its eggs, giving rise to immature stages. On the other hand, the coffee consumer exercises a selection process determining which products are established and which ones disappear definitively, mostly based on quality. An opposing force is used for the production and commercialization of specialty coffees. It is essential to study a mathematical model to understand how these two effects impact the coffee market and to determine conditions under a special coffee characterized by some attribute of quality differentiating it from standard coffee, can diversify the market. We study a mathematical model from the perspective of the Adaptive Dynamics. It allows for establishing conditions for the origin of diversity in demand for standard coffee in competition with specialty coffee when coffee production and the losses in production due to Coffee Berry Borer are considered.

2. The Model

The model describes competition of standard and special coffee considering coffee production and losses due to CBB:

$$\begin{cases} \dot{C} = rC\left(1-\frac{C}{k}\right) - \beta CM - hC \\ \dot{C}_b = \beta CM - (d+h)C_b \\ \dot{M} = \omega I - \mu M \\ \dot{I} = \epsilon\beta CM - (\omega+\delta) I \\ \dot{N}_1 = N_1\left[Q(q_1)hC + [1-Q(q_1)]\, hC_b - b(q_1)N_1 - f(q_1,q_2)N_2\right] \\ \dot{N}_2 = N_2\left[Q(q_2)hC + [1-Q(q_2)]\, hC_b - b(q_2)N_2 - f(q_2,q_1)N_1\right], \end{cases} \tag{1}$$

We denote derivatives with respect to **market time** t using dots. System (1) is subject to non-negative initial conditions at $t=0$, and defined in the set:

$$\Omega = \{0 \le C, C_b \le k, 0 \le M \le M_\infty, 0 \le I \le I_\infty, 0 \le N_1 \le N_{1\infty}, 0 \le N_2 \le N_{2\infty}$$

System (1) has nine constant solutions, but it is particularly interesting

$$\overline{E} = \left(\frac{\mu(\omega+\delta)}{\epsilon\beta\omega}, \frac{\mu r(\delta+\omega)}{\beta\epsilon\omega(h+d)}\frac{B_0-1}{B_0}, \frac{r}{\beta}\frac{B_0-1}{B_0}, \frac{r\mu}{\beta\omega}\frac{B_0-1}{B_0}, \frac{\mu(\omega+\delta)}{\epsilon\beta\omega}\frac{B_0Q(q_1)h+(B_0-1)(1-Q(q_1))r}{B_0b(q_1)}, 0\right),$$

when $B_0 = \frac{\beta k\omega\gamma\epsilon}{\beta hk\omega\epsilon+\mu\gamma(\omega+\delta)} > 1$. The system is considered at $\overline{E}$ before an innovation occur.

Coffee quality: The function $Q(q)$ is assumed to be a smooth sigmoid function from 0 (at $q=0$) to 1 (when $q\to\infty$), where q_i, for $i=1,2$ denotes two given (fixed) values of the coffee quality attribute q,

$$Q(q) = \frac{q^\alpha}{q_0^\alpha + q^\alpha}, \quad \alpha > 1, \tag{2}$$

where q_0 is a threshold quality that separates low quality coffee (if $q_i < q_0$, then $Q(q_i) < 1/2$, and promotes the use of bored grains) from high quality coffee (if $q_i > q_0$, then $Q(q_i) > 1/2$, and promotes the use of healthy grains).

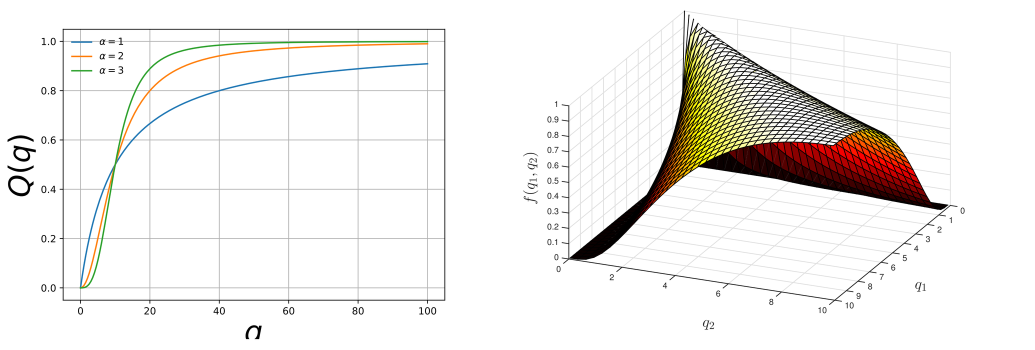

Figure 1: ***Left:*** *$Q(q)$ function with $q_0 = 10$, and $\alpha = 1$ (blue), $\alpha = 2$ (orange) and $\alpha = 3$ (green).* ***Right:*** *Competition function with $f_1 = 0.9$ and $f_2 = 1$.*

Qualities competion: Depends on the ratio q_1/q_2 and the technological parameters f_1 and f_2. Very diversified products compete weakly, products with greater technological "content" ($f_1 > 1$) tend to have a competitive advantage and products with less technological content ($0 < f_1 < 1$) are better competitors (see Fig. 1-right). Explicitly:

$$f(q_1,q_2) = \exp\left(\frac{\ln^2 f_1}{2f_2^2}\right)\exp\left(-\frac{1}{2f_2^2}\ln^2\left(\frac{f_1 q_1}{q_2}\right)\right)$$

If $f_1 = 1$, competition is symmetric and asymmetric if $f_1 \neq 1$. If high sensitivity to competition (small f_2), only very similar products compete, if low sensitivity (large f_2) then competition take place even between very different products [2].

3. Canonical equation and Evolutionary dynamics

Special coffee fitness function and invasion conditions: The relative variation rate of both coffee densities is given by:

$$\begin{aligned} \frac{\dot{N}_i}{N_i} &= g(C, C_b, M, I, N_1, N_2, q_1, q_2, \tilde{q}) \\ &= Q(\tilde{q})hC + [1-Q(\tilde{q})]\, hC_b - b(\tilde{q})N_i - f(\tilde{q}, q_i)\tilde{N}_i, \end{aligned}$$

for $i,j = 1,2$ and $i \neq j$ defines the **special coffee fitness function** that gives the relative variation rate of a low density of special coffee N_2. Function $\lambda(q_1,q_2)$ is an eigenvalue of the linearized system associated to (1) in $\overline{E}$ and determine the invasion conditions [1].

Evolutionary time is defined as $\varepsilon^2 t$ with $\varepsilon \to 0$, however, the dot still denotes time derivative [1]. Evolutionary dynamics of q is described by:

$$\dot{q} = \frac{1}{2}\mu(q)\sigma^2(q)\overline{N}_1(q)\frac{\partial\lambda}{\partial q_2}(q,q) = G(q) \tag{3}$$

evolutionary equilibria stability they are points satisfying $G(\overline{q}) = 0$ given by,

$$\overline{q}_0 = \left[-\frac{rq_0^\alpha}{h+d}\frac{B_0-1}{B0}\right]^{\frac{1}{\alpha}} \text{ and, } \overline{q}_{1,2} = \left[\frac{-F_1 \pm \sqrt{F_1^2 - 4F_2F_0}}{2F_2}\right]^{\frac{1}{\alpha}}$$

and they are locally asymptotically stable when,

$$G'(\overline{q}) < 0 \tag{4}$$

When $B_0 < 1$, the evolutionary equilibria $\overline{q}_0$ is a positive real number for every α, but $B_0 > 1$ is assumed in order to guarantee the presence of CBB at mature and immature stages, then $\overline{q}_0$ should be discarded.

Coexistence: In [1] it is shown that coexistence in (1) is possible for q_1 and q_2 close to $\overline{q}$ if:

$$\frac{\partial^2\lambda}{\partial q_1\partial q_2}(\overline{q},\overline{q}) < 0 \tag{5}$$

Divergence: two products that coexist, diverge in their attributes if:

$$\frac{\partial^2\lambda}{\partial q_2^2}(\overline{q},\overline{q}) > 0 \tag{6}$$

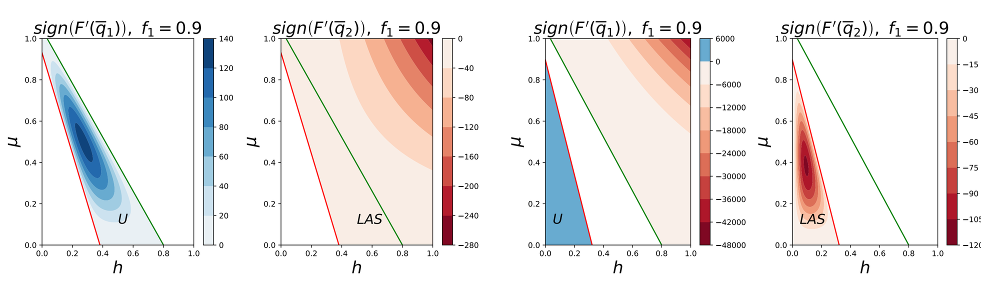

Figure 2: *Contour map of $G'(\overline{q}_i)$, for $i = 1$ and 2 in the $(h,\mu)-$plane. Color bar indicates the derivative's value and sign, left panels considering $f_1 < 1$ and $f_1 > 1$ in lower panels. The solid green curve corresponds to $B_0 = 1$. Parameter values are $q_0 = 10$, $\alpha = 3$, $f_2 = 0.3$, $k = nH\rho = 1642.5$, $r = 0.8$, $\beta = 0.05$, $\delta = 1/13.8$, $\omega = 1/7.95$, $\varepsilon = 0.02$, $d = 0.1$, $m = 1$; $\sigma = 1$*

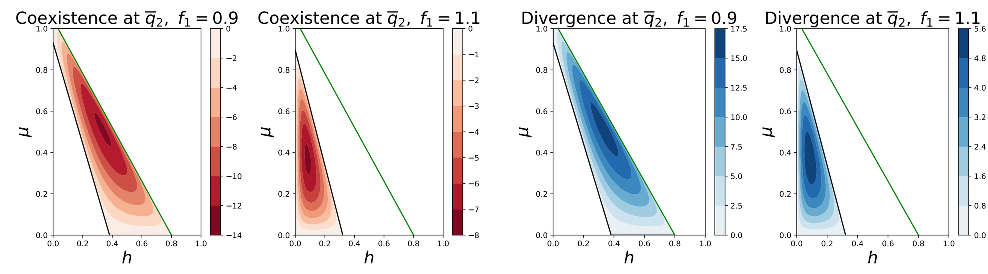

Figure 3: *The two contour map on left corresponds to coexistence condition (5) at $\overline{q}_2$ in $(h,\mu)-$plane with $f_1 = 0.9$ and $f_1 = 1.1$. The two right panels illustrate the divergence condition (6) for $f_1 = 0.9$ and $f_1 = 1.1$. Parameter are $q_0 = 10$, $\alpha = 3$, $f_2 = 0.3$, $k = nH\rho = 1642.5$, $r = 0.8$, $\beta = 0.05$, $\delta = 1/13.8$, $\omega = 1/7.95$, $\varepsilon = 0.02$, $d = 0.1$, $m = 1$; $\sigma = 1$,*

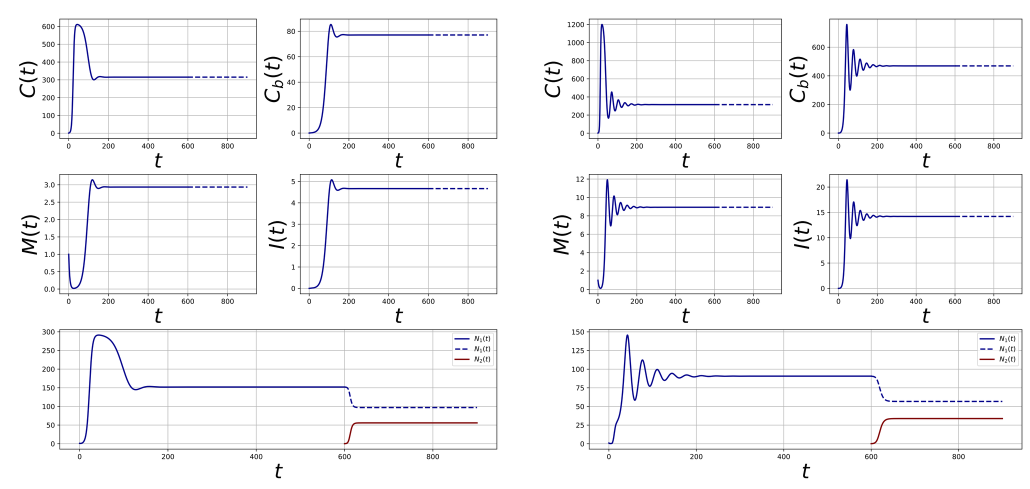

Figure 4: ***Left:*** *Simulation before and after innovation, considering $h = 0.5$ and $f_1 = 0.9$, then, $B_0 = 1.2246$, $q_1 = \overline{q}_1 = 27.2587$ and $q_2 = 1.1q_1$. Before the innovation (solid blue line), the simulation corresponds to $N_2 = 0$. After the innovation, the simulation corresponds to system (1) with initial conditions $\overline{E}$ and $N_2(0) = 0.1$. Evolutionary equilibrium is a Branching Point (BP), where market diversification arises.* ***Right:*** *Same scenario but $h = 0.2$, $\mu = 0.2$, $\beta = 0.05$ and $f_1 = 1.1$, then, $B_0 = 2.2645$, $q_1 = \overline{q}_1 = 4.8831$ and $q_2 = 1.1q_1$. The evolutionary equilibrium is a Branching Point (BP), where market diversification arises.*

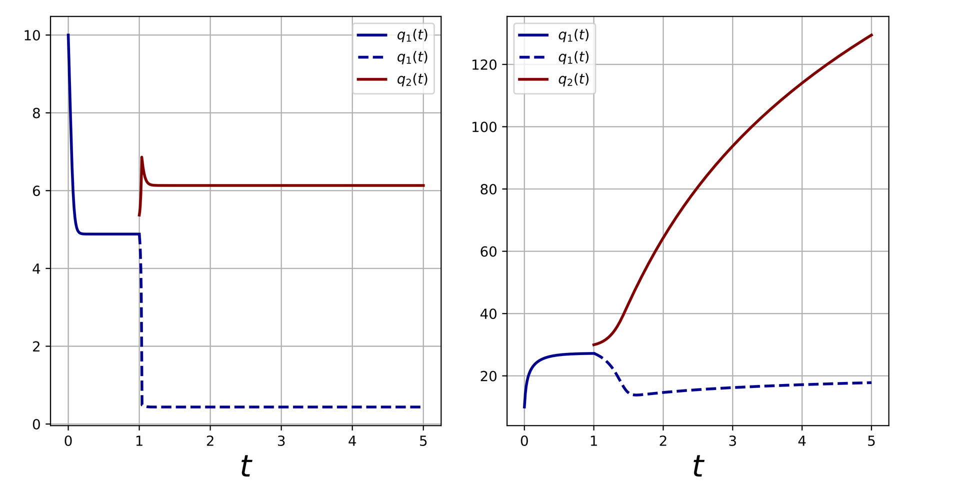

Figure 5: ***Left:*** *Simulation of (3) for q_1, before innovation (solid blue line) with initial condition $q_1(0) = 0.1$. After innovation, the $2-$dimensional canonical equation (3) for traits q_1 (dashed blue line) and q_2 (solid red line) is simulated, with $q_1 < q_2$, with $f_1 = 1.1$ and $h = 0.2$. This scenario correspond to a branching point. The parameter configuration leads to $B_0 = 2.2645$.* ***Right:*** *the same scenario but $f_1 = 0.9$ and $h = 0.5$. In this case $B_0 = 1.2246$.*

4. Conclusions

We formulate a deterministic model considering quality as a differentiating attribute of competing coffee types. We study the long term dynamics of quality traits from the perspective of Adaptive Dynamics, to establish conditions under which competition between standard and special coffee results in invasion, coexistence and divergence, to get the following classification:

- **Branching points (BP):** LAS equilibria in which the quality attribute can branch, which occurs when both conditions (5) and (6) are satisfied.
- **Terminal points (TP):** LAS evolutionary equilibria, but they are not branching points. At these points the evolution stops completely.
- **Bifurcation branching points (BBP):** Corresponds to border cases between branching points and terminal points.

The model allows for establishing conditions for the origin of diversity in demand for standard coffee in competition with specialty coffee when coffee production and the losses in production due to Coffee Berry Borer are considered.

5. Acknowledgments

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References

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